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Einstein's Gravitation for Machian Relativism of Nonlocal Energy-Charges

I.E. Bulyzhenkov-Widicker

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Abstract Tetrads require six metric bounds and energy-to-energy gravitation in the 1913 tensor generalization of the SR four-vector and the scalar four-interval. Only four energy-momentum components of the 1915 source equation can be relevant to flatspace gravitation of overlapping nonlocal carriers of energy-charges. New singularity-free metric equally works for the Einstein-Grossmann geodesic motion and for the r^{-4} elementary source in non-empty flatspace with the local time dilatation. The GR energy integral of the nonlocal radial (astro)carrier is finite and determines its active/passive gravitational charges. The SR reference for self-contained Einstein's relativity replaces the constant masses with their GR energies in the 1686 universal law of gravitation for the undivided world ensemble of overlapping radial matter. Gravitational/inertial energy-charges of nonlocal carriers depend on their global time-varying interactions with other elementary energy-charges that quantitatively address Machian relativism for gravitation and inertia. Electromagnetic waves change the gravitational/inertial energy-charge that can be tested in the Solar system. The non-empty space paradigm admits geometrization of the radial particle in the 1915 Einstein equation and suggests the similar field-energy nature for the distributed electric charge.

Keywords Non-empty space · Nonlocal energy-charge · 3D flatness · Gravitational four-potentials · Non-dual carriers · Unification · Continuous particle

1 Tensor Hilbert Variations with Six Metric Bounds for Four-Vectors

The space-time metric tensor in General Relativity (GR) is symmetrical and therefore the Hilbert [1] variations $\delta g^{\mu\nu}$ cannot admit more than ten independent variables for the 4D

I.E. Bulyzhenkov-Widicker
Institute of Spectroscopy, Troitsk, Moscow region 142092, Russia

I.E. Bulyzhenkov-Widicker (✉)
Department of Physics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada
e-mail: ibulyzhe@uottawa.ca

space-time manifold. Does this mean that GR metric fields in the 1915 Einstein equation [2, 3] are based on all ten degrees of freedom, which may be possible for pseudo-Riemannian mathematical spaces? The space-time-matter manifold should match all GR restrictions, including the vector geodesic conditions for the universal free fall. Therefore, the four-vector dynamics of probe particles may result in additional Euler-Lagrange bounds for tensor variables of real matter with physical restrictions.

Again, components of metric tensors $g_{\mu\nu}$ and $g^{\mu\nu}$ should universally work for active (source) matter in the 1915 tensor equation and for passive (probe) particles in the GR four-vector relations. It is essential that the 1913 Einstein-Grossmann tensor project for the geodesic variations [4] relates only four metric components $g_{o\mu}$ to the observable 3D force,

$$f_i = -\frac{m\sqrt{g_{oo}}}{\sqrt{1-v^2}}[\partial_i(-g_{oo}^{-1/2}) + v^j[\partial_i(g_{oj}/g_{oo}) - \partial_j(g_{oi}/g_{oo})]], \tag{1}$$

in any constant field with $\partial_o g_{\mu\nu} = 0$, for example [5].

Why are the six components g_{ij} ($i, j, a, b = 1, 2, 3$ and $\mu, \nu, \alpha, \beta = 0, 1, 2, 3$) not relevant to observations and measurements in the 3D laboratory with a 'curved' 3D metric tensor $\gamma_{ij} \equiv g_{oi}g_{oj}g_{oo}^{-1} - g_{ij}$? Does 1916 Schwarzschild-Droste warping [6, 7] of γ_{ij} really exist in practice or is it a mathematical flaw from the ill-modeled point source of gravity? Below we consider the inherent metric symmetries in the 1913 Einstein-Grossmann generalization of Special Relativity (SR) and then restart the Hilbert variations with extra Euler-Lagrange bounds. Four independent fields in the metric tensor can specify for such a generalization only four source-field relations in the Einstein equation.

Let us consider the GR four-momentum $P_\mu \equiv mV_\mu \equiv mg_{\mu\nu}dx^\nu/ds$ of a scalar mass m through the known tetrad formalism [5] with $g_{\mu\nu} \equiv \eta_{\alpha\beta}e_\mu^{(\alpha)}e_\nu^{(\beta)}$, $e_\mu^{(o)} = \{\sqrt{g_{oo}}; -\sqrt{g_{oo}}g_i\}$, $e_\mu^{(b)} = \{0, e_i^{(b)}\}$, and $g_i = -g_{oi}/g_{oo}$. We compare spatial components of 'curved' V_μ and 'plane' $V_{(\beta)} = \{(1 - v_{(b)}v^{(b)})^{-1/2}; -v_{(b)}(1 - v_{(b)}v^{(b)})^{-1/2}\}$ four-velocities, $-(\sqrt{g_{oo}}g_i + v_i) \times (1 - v_iv^i)^{-1/2} \equiv V_i \equiv e_i^{(\beta)}V_{(\beta)} \equiv e_i^{(o)}V_{(o)} + e_i^{(b)}V_{(b)} \equiv -(\sqrt{g_{oo}}g_i + e_i^{(b)}v_{(b)})(1 - v_{(b)}v^{(b)})^{-1/2}$, in order to revisit mathematical possibilities for the 1913 Einstein-Grossmann generalization. In our view, a reasonable generalization V_μ of the SR four-vector $V_{(\beta)}$ requests universal relations $v_iv^i = v_{(b)}v^{(b)}$ and $v_i = e_i^{(b)}v_{(b)} = \delta_i^{(b)}v_{(b)}$ between the 'curved' 3-velocities, $v_i = \gamma_{ij}dx^j/d\tau$, and the 'plane' 3-velocities, $v_{(b)} = \delta_{ab}dx^{(a)}/d\tau$, where $d\tau \equiv \sqrt{g_{oo}} \times (dx^o - g_idx^i)$.

These universal relations for the spatial tetrad with the Kronecker tensor, $e_i^{(b)} = \delta_i^{(b)}$, reveal that the three-space metric tensor $\gamma_{ij} \equiv g_{oi}g_{oj}g_{oo}^{-1} - g_{ij} = \delta_{ij}$ is irrelevant to gravitational fields in the 1913 metric approach to the SR interval of a probe particle. GR tetrads justify that particles within differently warped 4D manifolds always hold Euclidean 3D subgeometry in gravitational fields. By way of illustration, any 2D sheet of paper could keep its parallel lines without intersections no matter how one would warp this sheet in 3D space. A 4D volume determinant $\sqrt{-g} \equiv \sqrt{g_{oo}|\gamma_{ij}|} = \sqrt{g_{oo}}$ for real space-time-matter depends only on one metric component. Spatial flatness for four-vector relativistic motion of probe particles has to be unconditionally preserved in all GR solutions.

The flatspace Universe takes many computational advantages. One may split, for example, the covariant four-momentum, $P_\mu = e_\mu^{(\alpha)}V_{(\alpha)} = e_\mu^{(b)}P_{(b)} + e_\mu^{(o)}P_{(o)} = (e_\mu^{(b)}P_{(b)} + \delta_\mu^{(o)}P_{(o)}) + (e_\mu^{(o)} - \delta_\mu^{(o)})P_{(o)} \equiv P_{(\mu)} + U_\mu$, into the mechanical part, $e_\mu^{(b)}P_{(b)} + \delta_\mu^{(o)}P_{(o)} = \delta_\mu^{(\alpha)}P_{(\alpha)} = P_{(\mu)}$ (because $e_o^{(b)} = 0$ and $e_i^{(b)} = \delta_i^{(b)}$), and the gravitational contribution $U_\mu \equiv (e_\mu^{(o)} - \delta_\mu^{(o)})P_{(o)}$. From here the GR gravitational energy U_μ of a probe particle takes only four components and depends on the SR mechanical energy $P_{(o)} = m/\sqrt{1-v^2} = P_o/\sqrt{g_{oo}}$ and four gravitational potentials $G_\mu \propto U_\mu$ in the 'time' tetrad $e_\mu^{(o)} = \delta_\mu^{(o)} + U_\mu P_{(o)}^{-1}$.

Now one may infer from (1) with the potentials $(-g_{oo}^{-1/2})$ and g_{oi}/g_{oo} that the Einstein-Grossmann tensor generalization should tend to describe energy-to-energy relativity because only the total GR energy P_o, rather than the particle mass m, may pretend to match these field potentials at arbitrary relativistic velocities, with $0 \le v^2 < 1$. In this way, the source energy content also ought to represent the active gravitational charge for interactions with the passive energy-charge P_o. We denote the total GR energy P_o as a gravitational/inertial charge by assuming its conservation in constant fields and Machian changes [8] in time-varying fields, when $\partial_o g_{\mu\nu} \neq 0$. The general source equation should determine four field potentials $G_\mu \equiv U_\mu / P_o$ for this positively defined charge, $P_o > 0$. These potentials contribute to the particle four-momentum, $P_\mu = P_{(\mu)} + P_o G_\mu$, to the GR tetrad

$$e_\mu^{(\beta)} = \delta_\mu^{(\beta)} + \delta_o^\beta U_\mu P_{(o)}^{-1} = \delta_\mu^{(\beta)} + \delta_o^\beta G_\mu/(1 - G_o) \tag{2}$$

and to GR metric tensors $g_{\mu\nu} \equiv \eta_{\alpha\beta} e_\mu^{(\alpha)} e_\nu^{(\beta)}$ and $g^{\mu\nu}$, with $g_{\mu\nu} g^{\mu\rho} = \delta_\nu^\rho$ and $g_{oi} g_{oj} g_{oo}^{-1} - g_{ij} = \delta_{ij}$,

$$\begin{cases} g_{oo} \equiv e_o^{(o)} e_o^{(o)} = (1 + U_o P_{(o)}^{-1})^2 = 1/(1 - G_o)^2, \\ g_{oi} \equiv e_o^{(o)} e_i^{(o)} = (1 + U_o P_{(o)}^{-1}) U_i P_{(o)}^{-1} = g_{oo} G_i, \\ g_{ij} \equiv e_i^{(o)} e_j^{(o)} - \delta_{ab} e_i^{(a)} e_j^{(b)} = U_i U_j P_{(o)}^{-2} - \delta_{ij} = g_{oo} G_i G_j - \delta_{ij}, \\ g^{oo} = (1 - G_o)^2 - \delta^{ij} G_i G_j, \quad g^{oi} = \delta^{ij} G_j, \quad g^{ij} = -\delta^{ij}. \end{cases} \tag{3}$$

Here we used $P_{(o)} = P_o/\sqrt{g_{oo}}$ in order to find that $\sqrt{g_{oo}} = 1 + \sqrt{g_{oo}} U_o P_o^{-1} = 1/(1 - U_o P_o^{-1}) = 1/(1 - G_o)$ for the most general kind of motion. Notice for this flatspace generalization of SR dynamics that $G_o \equiv 1 - g_{oo}^{-1/2} < 0$, $G_i \equiv -g_i$, and that the generalized energy, $P_o = m\sqrt{g_{oo}}/\sqrt{1 - v^2} = m/\sqrt{1 - v^2}(1 - U_o P_o^{-1}) = (m/\sqrt{1 - v^2}) + U_o$, represents the strong-field gravitational energy, $U_o = P_o G_o < 0$, separately from the SR mechanical energy $m/\sqrt{1 - v^2} > 0$.

The non-linear potential G_μ in (1–3) is not a four-vector in pseudo-Riemannian space-time of any considered particle, because $U_\mu \propto G_\mu$ is only a part of the particle four-vector P_μ. Furthermore, six degrees of freedom (from ten possible degrees in symmetrical rank 2 tensors) are strictly bound in $g_{\mu\nu}$ for real matter by four-vector dynamics with mandatory flatness, $g_{oi} g_{oj} g_{oo}^{-1} - g_{ij} = \delta_{ij}$. These inherent symmetries in the pseudo-Riemannian metric formalism match the GR geodesic relations and, ultimately, the Principle of Equivalence. Therefore, these six metric bounds are conceptual for physical GR solutions, including the particle four-interval $ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = d\tau^2 - dl^2$ with the universal space subinterval $dl^2 \equiv \gamma_{ij} dx^i dx^j = \delta_{ij} dx^i dx^j$ and the local time element

$$d\tau^2 \equiv g_{oo}(dx^o - g_i dx^i)^2 = \frac{(dx^o + G_i dx^i)^2}{(1 - G_o)^2} \tag{4}$$

with the gravitational Sagnac effect for $G_i \neq 0$.

Three bounds $g_{ij} = g_{ji}$, three bounds $g_{oi} = g_{io}$, and six bounds $g_{oi} g_{oj} = g_{oo}(g_{ij} + \delta_{ij})$ or $g^{ij} = -\delta^{ij}$ may admit only four independent relations in sixteen 1915 equations for the tensor source. What are these four source-field relations for the flatspace generalization of the SR interval? Euler-Lagrange variations of the Hilbert action [1, 5] $S = \int [\Lambda - (g^{\mu\nu} R_{\mu\nu}/2\kappa)]\sqrt{-g}\, d\Omega$ with respect to the non-vector potentials G_μ would not result in Lorentz-invariant equations with a definite tensor structure. However, components of the metric tensors (3) smoothly depend on these four-component potentials. Seven contravariant variations δg^{io}, δg^{oi}, and δg^{oo} with three bounds $g^{oi} = g^{io}$ can result only in four Euler-Lagrange equations for the GR energy-momentum tensor $T_{\mu o}$, while nine other variations vanish, $\delta g^{ij} \equiv 0$ due to (3). In other words, nine tensor equations for the stress components $T_{ij} = T_{ji}$ cannot arise in flatspace source physics (with the twelve metric symmetries for passive matter) from the Hilbert variations.

One may also employ the general variational link $g_{\mu\nu}\delta g^{\rho\nu} = -g^{\rho\nu}\delta g_{\mu\nu}$ and tetrads with the flatspace bounds $e^{(b)}_{\mu} = \delta^{(b)}_{\mu}$ (or $\delta e^{(b)}_{\mu} = 0$) in $\delta g_{\mu\nu} \equiv \eta_{\alpha\beta} e^{(\beta)}_{\nu}\delta e^{(\alpha)}_{\mu} + \eta_{\alpha\beta} e^{(\alpha)}_{\mu}\delta e^{(\beta)}_{\nu} = \eta_{o\beta} e^{(\beta)}_{\nu}\delta e^{(o)}_{\mu} + \eta_{\alpha o} e^{(\alpha)}_{\mu}\delta e^{(o)}_{\nu} = 2\eta_{o\beta} e^{(\beta)}_{\nu}\delta e^{(o)}_{\mu}$. Finally, the variations with respect to field-dependant components of the flatspace tetrad $e^{(\alpha)}_{\mu}$ also result in four equations for sources and their fields. One can find for mixed covariant-contravariant components that

$$\left(g^{\mu\lambda}g^{\nu\rho}T_{\lambda\rho} - \frac{g^{\mu\lambda}g^{\nu\rho}R_{\lambda\rho}}{\kappa} + \frac{g^{\mu\nu}g^{\lambda\rho}R_{\lambda\rho}}{2\kappa}\right)\frac{\sqrt{-g}}{2}2\eta_{o\beta}e^{(\beta)}_{\nu} \equiv T^{\mu}_{o} - \frac{1}{\kappa}\left(R^{\mu}_{o} - \frac{1}{2}\delta^{\mu}_{o}R\right) = 0, \tag{5}$$

where $\sqrt{-g}\eta_{o\beta}e^{(\beta)}_{\nu} = \sqrt{-g}\eta_{oo}e^{(o)}_{\nu} = \sqrt{g_{oo}}e^{(o)}_{\nu} = g_{o\nu}$, $T^{\mu}_{o} = g^{\mu\lambda}T_{\lambda o}$, $R^{\mu}_{o} = g^{\mu\lambda}R_{\lambda o}$, and $R = g^{\lambda\rho}R_{\lambda\rho}$.

Fields of moving sources in (5), known with $\kappa = 8\pi G/c^2 = 1.86 \times 10^{-26}$ m/kg as Einstein's equations [2, 3] for the energy-momentum tensor density T^{μ}_{o}, contain only first partial derivatives with respect to the world-time $t \equiv x^o/c$, with $c = 1$. Therefore, initial conditions for field solutions should be defined only for four metric tensor components or for four potentials G_{μ}, rather than for both metric potentials and their time derivatives [5] in the nine omitted Einstein's equations for the spatial stress-tensor T^{i}_{j}. Below we find from (5) the law for gravitational interactions of the introduced energy-charges P_o instead of the conventional attraction of masses.

2 Geometrization of the Continuous Particle in Non-Empty Flatspace

Flat space metric solutions of the four-component source equation (5) would never coincide with Schwarzschild's solution or another 'point-source and empty space' modeling of physical reality. Machian energy-to-energy relativity (1–4) suggests the nonlocal active/passive charge P_o with the radial energy-density T_{oo} integrated into the nonlocal astrostructure of the particle field. The integration of the particle into the space structure of its field was assumed by Einstein: 'We could regard matter as being made up of regions of space in which the field is extremely intense . . . . There would be no room in this new physics for both field and matter, for the field would be the only reality', translation [9]. An extended particle has not been yet introduced analytically into the classical field theory, which relies instead on the Dirac delta-function for operator relations between spatially separated field and matter points. However, the electric self-energy density of the extended electron can locally represent the elementary charge distribution in the analytical Poisson equation [10]. Therefore,, the controversial formal assignment of different summands in the Einstein field equation (5) to different space arguments for the traditional choice of the point source may be considered as our mathematical motivation to replace the point particle with the radial energy-density distribution. Our physical motivation to look at space overlap of continuous particles replicates Newton's original approach to the gravitational action-at-a-distance through stresses in the material inter-bodies medium, which in 1686 was introduced as ether or 'God's sensorium'.

Let us revisit the Einstein-Grossmann probe material point in a static central field of a gravitational source with the motionless active mass M_a and energy E_a. The passive GR energy E_p of any probe (passive) mass m_p,

$$E_p \equiv \frac{m_p\sqrt{g_{oo}}}{\sqrt{1-v^2}} = K + U_o, \tag{6}$$

incorporates the Special Relativity (SR) mechanical energy K and the so far undefined gravitational energy U_o. We may consider the source and the test particle as non-point radial objects with the spherical symmetry of their material densities. Then the gravitational energy of mutual interactions may depend on a radial distance, $U_o = U_o(r)$, between geometrical centers of two such objects. However we are not going to employ Newtonian references from the mass-to-mass gravitation. We plan to derive energy-to-energy self-references from the SR mechanical relation $K \equiv m_p/\sqrt{1-v^2}$. In this way Mach-Einstein relativity for nonlocal energy-charges may be discussed as a self-contained theory of gravitation.

First we redesign the GR metric component $\sqrt{g_{oo}}$ of the pseudo-Riemannian metric tensor $g_{\mu\nu}$ in (6) with the following equalities,

$$\sqrt{g_{oo}} \equiv \frac{K\sqrt{1-v^2}}{m_p} + \frac{U_o\sqrt{g_{oo}}}{E_p} \equiv \frac{K\sqrt{1-v^2}}{m_p c^2(1-U_oE_p^{-1})} \equiv \frac{1}{1-U_oE_p^{-1}}, \tag{7}$$

based on the SR reference for the mechanical energy $K = K(r)$. Only this metric component contributes to the GR interval equation $ds^2 = g_{oo}dt^2 - dl^2$ for a static field, when $g_{oi} = 0$,

$$ds^2 = \frac{dt^2}{(1-U_oE_p^{-1})^2} - dl^2 = g_{oo}dt^2(1-v^2) = \left(\frac{m_p g_{oo} dt}{E_p}\right)^2. \tag{8}$$

The continuous astrosource (or astroparticle) possesses the integral energy-momentum P_μ and the tensor energy-momentum density $T_{\mu o}$, associated with the gravitational equation (5). This tensor equation is valid in all coordinate systems and we use the rest-frame system of references with the static self-field without loss of generality. Here the energy-momentum tensor has only one non-vanishing component, $T_{oo} = g_{oo}T^o_o$, related to the energy density of sources. Recall, Einstein maintained that both sides of his gravitational equation should be considered at field points. Therefore, we also may assume a radial distribution of the astroparticle tensor density $T_{oo}(r)$, which generates a central gravitational field with spherical symmetry in (5) and (7).

By integrating the distributed particle into its spatial field structure, one accepts that curved space-time is doubly warped by the local elementary energy (by both the particle-energy density and the field-energy density). Such a non-dual unification of the continuous particle and its field should lead to a complete geometrization of the bi-fractional (particle + field) nonlocal carrier of energy. The point particle term vanishes in the Hilbert action for non-empty space as well as the point source peculiarity $T_{o\nu}$ next to the Einstein curvature $G_{o\nu} = R_{o\nu} - g_{o\nu}R/2$ in (5). In other words, the non-empty space paradigm 'simplifies' the Einstein equation (5) for the case of continuous particles,

$$g^{\mu\nu}R_{o\nu} - \delta^\mu_o R/2 = 0, \tag{9}$$

that automatically results into the local energy-momentum conservation $G^\mu_{o;\mu} \equiv 0$ for the unified particle-field carrier. Now the zero Einstein curvature, $G^\mu_o \equiv g^{\mu\nu}G_{o\nu} = 0$ (i.e. $g^{oi}R_{oo} = \delta^{ij}R_{oj}$, $g^{oo}R_{oo} = -\delta^{ij}R_{ij}$), and the non-zero Ricci curvatures, $R_{o\nu}$ and R, incorporate both the gravitational field and its continuous particle-source component (or distributed T_o^μ).

Such a geometrization with zero Einstein curvature for the unified particle-field matter corresponds, in principle, to the 1876 'space-theory of matter' [11] and to the 1686 ether theory for gravitating matter [12]. Contrary to zero Riemann curvature of unwarped 4D manifolds, zero Einstein curvature does not mean the absence of locally bounded particle and fields densities in warped space-time. One can find for $G_o^o = 0$, for example, the local mass-energy balance in $g^{o\nu} R_{o\nu} = R/2$ by relating the Ricci scalar curvature $R \equiv g^{\mu\nu} R_{\mu\nu} \equiv 8\pi G(\mu_a + \mu_p)$ to the particle-source (active) energy density μ_a plus the generated field (passive) mass-energy density μ_p of the unified continuous carrier. The Ricci tensor $R_{\mu\nu}$ can be easily found for the static metric, when $g_{oi} = 0$ and $\partial_o g_{oo} = 0$. Indeed, the distributed static energy warps $g_{oo}(r) = 1/g^{oo}(r) = [1 - U(r)E_p^{-1}]^{-2}$ for probe energies E_p with, as so far, an unreferenced potential energy $U_o(r)$. Only two warped connections $\Gamma^i_{oo} = \partial_i g_{oo}/2$ and $\Gamma^o_{io} = \partial_i g_{oo}/2g_{oo}$, when $\partial_o g_{\mu\nu} = 0$ and $U_i = 0$, define $R^o_o = g^{oo} R_{oo} = g^{oo}(\partial_i \Gamma^i_{oo} - \Gamma^j_{oo}\Gamma^o_{oj}) = [-\partial_i^2 \ln(g_{oo}^{-1/2}) + (\partial_i \ln(g_{oo}^{-1/2}))^2]$ and $R = g^{oo} R_{oo} + g^{ij} R_{ij} = g^{oo}(\partial_i \Gamma^i_{oo} - \Gamma^j_{oo}\Gamma^o_{oj})$ - $\delta^{ij}(-\partial_j \Gamma^o_{oi} - \Gamma^o_{io}\Gamma^o_{jo}) = 2R^o_o$, with

$$\frac{R}{8\pi G} = \frac{g^{oo} R_{oo}}{4\pi G} = \frac{-\nabla \mathbf{w} + \mathbf{w}^2}{4\pi G}. \tag{10}$$

We related local active/passive energy densities to the scalar Ricci curvature R and introduced the post-Newtonian field intensity $\mathbf{w} \equiv -\nabla W$ through the static potential of the active charge $W \equiv -\ln(1/\sqrt{g_{oo}})$ for passive energy-charge densities (one may equally define $W^* \equiv -W > 0$ for electromagnetic-type attraction of negative passive charges $(-E_p) < 0$ by a positive active charge $E_a > 0$). The summand $-\nabla \mathbf{w}/4\pi G \equiv \mu_a$ can be associated with the particle-source (active) energy density μ_a for the unified carrier, while the next summand with the field (passive) energy density $\mu_p \equiv \mathbf{w}^2/4\pi G$. These local energy densities can be integrated over all material space and they define total active/passive mass-energies of elementary matter.

Below we verify that field and particle fractions of the unified carrier equally contribute to the scalar Ricci curvature, because $(\nabla \mathbf{w})^2 = -\nabla \mathbf{w}$ or $\mu_p = \mu_a$, that equalizes passive and active mass-energy integrals $\int \mu_p d^3x = \int \mu_a d^3x \equiv E_a$ of the nonlocal elementary carrier. First we analyze the radial particle (active) contribution in (10),

$$\begin{aligned} E_a &= \frac{1}{4\pi G} \int 4\pi r^2 dr \nabla(-\mathbf{w}) = \frac{1}{G} \int dr \partial_r [-r^2 \partial_r \ln(g_{oo}^{-1/2})] \\ &= -\frac{r^2}{G} \partial_r \ln(g_{oo}^{-1/2}) \bigg|_o^\infty = \frac{r^2 \partial(U_o E_p^{-1})}{G(1 - U_o E_p^{-1})} \bigg|_o^\infty. \end{aligned} \tag{11}$$

We use in (11) the metric tensor (7) based on the SR mechanical references. Now, by solving (11) with respect to $U_o(r)$, one can derive the energy-to-energy attraction law

$$U_o(r) = \frac{G E_a(-E_p)}{r}. \tag{12}$$

This universal law specifies the unreferenced metric component $g_{oo} = [1 + r_o/r]^{-2}$ and smooth static metric $d^2s = g_{oo} dt^2 - \delta_{ij} dx^i dx^j$ with $r_o \equiv G E_a$ in the self-contained SR-GR theory of gravitation.

We did not take Newtonian references for strong field gravitation (9–12). Mach anticipated time-varying charges (GR energies in our approach) in the universal gravitational law. Interactions in the self-referenced SR-GR theory depend conceptually on attractions of passive energy charges by active energy charges, rather than by mutual attractions of constant scalar masses (like it might seem from astronomical observations of non-relativistic bodies). This approach can quantitatively address Mach's ideas [8] regarding variations of passive/inertial and active gravitational charges ($E_p \neq const$ and $E_a \neq const$ when $\partial_o g_{\mu\nu} \neq 0$) in the 1686 Newton law for gravitation with the constant masses.

Contrary to the empty-space model with point sources for Schwarzschild's solution, the Newton-Clifford material space is filled or continuously charged everywhere by both equal active and passive mass-energy densities $\mu_a = \mu_p = r_o^2/4\pi G r^2(r_o + r)^2$. Our geometrization of non-dual matter maintains the r^{-4} radial carrier with continuous particle and field fractions over the entire Universe, with

$$\begin{cases} 4\pi\mu_a(r) = -\nabla\mathbf{w}/G = E_a r_o/r^2(r+r_o)^2 = \mathbf{w}^2/G = 4\pi\mu_p(r), \\ \mathbf{w}(r) = -\nabla W(r) = -GE_a\hat{\mathbf{r}}/r(r+r_o), \\ W(r) = -\ln[(r+r_o)/r], \\ r_o/G = \int \mu_a(r)d^3x = E_a. \end{cases} \tag{13}$$

The continuous particle-field carrier (which generates the post-Newtonian logarithmic potential W) is in agreement with the well-known concerns of Einstein regarding point particles in his 1915 equation: 'it resembles a building with one wing built of resplendent marble and the other built of cheap wood'. The r^{-4} material 'tails' of elementary particles, overlapping on microscopic, macroscopic, and megascopic scales, occupy the total Universe in full accord with the 'absurd' Newtonian ether. It is simply unbelievable that the mathematician inferred in 1686 that the observable world is nonlocal and that space is filled everywhere by gravitational matter of globally interacting bodies.

Classical Electrodynamics tends to bypass Newtons's and Clifford's ideas regarding the non-empty space paradigm. However, the exact radial solution to Maxwell's equations [10],

$$\begin{cases} 4\pi\rho(r) = er_e/r^2(r+r_e)^2 = \mathbf{w}_e^2(r)r_e/e = \nabla\mathbf{w}_e(r), \\ \mathbf{w}_e(r) = -\nabla W_e(r) = e\hat{\mathbf{r}}/r(r+r_e), \\ W_e(r) = (e/r_e)\ln[(r+r_e)/r], \\ e^2/r_e = (e/r_e)\int\rho(r)d^3x = \int d^3x\mathbf{w}_e^2(r)/4\pi = \int\rho(r)W_e(r)d^3x, \end{cases} \tag{14}$$

analytically justifies non-empty space. The Maxwell electron is distributed over the entire Universe with half of its negative charge, $e \equiv \int_o^\infty 4\pi r^2\rho(r)dr = -e_o < 0$, within the microscopic radial scale r_e. Notice, that the electron's charge density $\rho(r)$ is locally proportional to the electron's field energy density and $\mathbf{w}_e^2/4\pi - e\nabla\mathbf{w}_e/4\pi r_e = e(\rho_f - \rho_{ch})/r_e = 0$ is massless in the Ricci scalar (10) of the unified nonlocal carrier. Therefore, gravitational and electric charges may have the similar energy nature, $r_e = r_o$, in the considered non-empty space paradigm.

Newton-Clifford material space with gravitational ether, specified by the r^{-4} astrodistribution (13), differs in principle from the Schwarzschild 'point source-empty space' modeling of physical reality. And Birkhoff's theorem for empty and curved 3D spaces cannot be responsible for self-consistent metric solutions of the Einstein equation for non-empty space. Moreover, our static field metric (8), $ds^2 = dt^2(1 + r_o r^{-1})^{-2} - dl^2$, for the universal attraction (12) between passive and active energies has been derived to criticize ad hoc dogmas of the empty world space and point matter peculiarities in empty space. This new metric for strong radial fields is free from Schwarzschild singularities criticized by Einstein [13].

The nonlocal inertial and gravitational charge P_o is the first integral of motion only in static fields of fixed 'distant' bodies. Realistic non-stationary solutions $P_o(t) \neq const$ for many-body astrophysical systems and their possible observations could quantitatively justify the original Mach's idea about relativity of inertia. Machian relativity of gravitation can be verified, for example, in the Solar system due to the known Sun's luminosity, 3.85×10^{26} $W = \dot{P}_o(t)$, and the electromagnetic radiation decay of the Sun's energy-charge, $P_o(t) \approx 1.99 \times 10^{30}$ kg $\times\ 9 \times 10^{16}$ $\mathrm{m}^2/\mathrm{s}^2$. Electromagnetic energy losses affect Machian energy-charges even with a fixed number of baryons. These losses may result in measurable annual inflation of Keplerian semi-major axes, $\dot{r}(t)/r(t) = -\dot{P}_o(t)/P_o(t)$, of revolving planets. The solar wind carriers massive plasma and such energy-mass losses equally contribute to the planet orbit inflation in Mach-Einstein's energy-to-energy relativity and in Newton's mass-to-mass gravitation. But it is a specific point for original Machian relativism that black body radiation or body temperature variations result in post-Newtonian corrections under the Einstein geodesic motion in weak fields.

One can predict the Neptune-Sun semi-axis (4.5×10^{12} m) enhancement of above 50 m per each Neptune revolution (163.7 Earth's years) and above one cm per year for the Earth-Sun average distance (1.5×10^{11} m) exclusively due to the 'massless' wave radiation by the Sun. Temperature-varying modifications of known experiments with asymmetrical Cavendish-Eötvös scales [14] can be considered for laboratory tests of the variable energy-charge in Machian gravitational interactions. Moreover, the energy-to-energy gravitation law (12) suggests, for example, that massless photons with their finite gravitational energy-charges should be counted in the current search of dark matter.

There are many physical reasons to follow Mach relativism and the Newton-Einstein directive toward continuous sources in the classical field theory. In particular, the r^{-4} radial particle relieves this theory from the electric energy divergence of the point charge. And clear classical notions for the continuous particle can be very useful for casual interpretation of quantum 'dices' in the nonlocal [15] Universe. By closing, the gravitational four-potentials G_μ in flatspace may enable quantization of gravity and its unification with vector electrodynamics.

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